Studies of the Module Alignment of the ATLAS Pixel Detector Using Simulated Cosmic Data

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Abstract

The Pixel Detector (PIXEL) is the innermost tracking detector of the ATLAS experiment at the Large Hadron Collider. It is built closest to the interaction point of the two counter rotating proton beams, and its spatial resolution is essential to the accurate reconstruction of collision events. Misalignments of the PIXEL silicon wafer modules from their nominal position significantly impair the resolution. A method for assessing the relative misalignment of the modules of the three PIXEL barrel layers, based on analysis of straight line tracks, is furnished. The misalignment model assumes that the modules are rigid bodies with 6 degrees of freedom represented by 6 alignment constants. Two approaches are used to calculate the constants, both of which are based on parametrizing residuals - departures from the expected intersections of the particle trajectory with the detector modules. In this study it is assumed that modules in the outermost barrel layer (L2) can be used as reference and the rest of the modules can be aligned with respect to L2. The residuals calculated relative to a line fitted to the clusters on L2 are expressed in terms of the alignment constants to the first order. The constants are extracted independently - by fitting histograms of the residuals (histogram-based approach), and simultaneously - by minimizing a merit function (χ^2-based approach). Both approaches are applied to simulated cosmic data representing muons passing through the PIXEL. Only modules that contain more than 50 hits are considered for alignment. The constant sets recovered by both approaches are consistent with the true values used to generate the simulated data. The calculated constants are used to recalculate the spatial resolution of the PIXEL, which yields improvements of up to 40% in local x and up to 3% in local y resolution. Limitations of the model, including sensitivity, effects of noise and convergence upon iteration, are also investigated. As a result, the model is considered to be a viable approach to improving the PIXEL resolution within its limitations, provided that tests with real data confirm the findings.

Introduction

The Large Hadron Collider (LHC) at CERN is a circular particle accelerator expected to operate at 14 TeV. The ATLAS experiment at LHC is built around one of four interaction points of two counter-rotating proton beams. The Pixel Detector (PIXEL) is the innermost tracking detector of ATLAS. The accuracy in the spatial resolution it provides is crucial to the reconstruction of particle trajectories originating from the collision point. The intricate geometry of the PIXEL barrel can be crudely modeled as three nested right circular cylinders, the largest of which has a diameter of 0.4 m and a length of 1.6 m. Approximately 80 million individually controlled pixels, with dimensions 50 μm by 400 μm, are evenly distributed in 1744 silicon wafer modules. The 3 barrel layers hold 1456 modules, while the remaining 288 modules are in two endcaps at each end of the barrel. The Pixel Detector is assembled as close to specifications as possible, however, small deviations in the module positions are present. To ensure that the performance requirements on ATLAS are met, it is important that the actual spatial arrangement of the PIXEL modules is sufficiently accurate.

The following study examines a method for measuring the relative misalignment of the barrel modules via a histogram-based alignment approach and a χ^2 minimization of parametrized residuals. The algorithms can be applied to particle track data to yield sets of 6 constants per module, which can be used to recalculate the spatial resolution of the PIXEL. Results obtained by applying the procedure to cosmic muon simulation data are discussed.

Method

Geometry and Notation

Intersections of particle trajectories with the detector surfaces, *hits*, are specified by right-handed Cartesian coordinate systems. The origin of the global frame, defined by ATLAS, is at the center of the detector, the nominal beam collision point. The global x-axis points toward the center of the accelerator (LHC) ring and the global y axis points up. The barrels are oriented along the global z-axis, which is tangent to the beam in the plane of the LHC ring. Each module defines a local frame with its origin at the center of the module. The module lies in the xy plane of the local frame. The local z-axis points outward from the global z-axis and the local x-axis points in the direction of increasing global ϕ, the azimuthal cylindrical coordinate.

With the above definitions, the structure is further specified by the following. The barrel layers, denoted by L0, L1 and L2, are composed of 22, 38 and 52 staves, respectively (Fig. 1). A stave contains 13 modules labeled by η - an index ranging from -6 to 6. Each module is rotated by $20°$ about its local y axis and by $-\mathrm{sgn}(\eta)1.1°$ about its local x axis.

Misalignment Model Description

The readout from the PIXEL as a particle passes through a module contains information that can be used to recover the position of the *recorded* hit, $(x_r, y_r, 0)$. In the model it is assumed that tracks - collections of hits along the particle path - are straight lines. The intersection of a track with the plane representing the nominal position of a module is the *expected* hit, with coordinates $(x_e, y_e, 0)$. The differences between the coordinates of the recorded and the expected hits are defined as the *residuals* $\Delta x \equiv x_r - x_e$ and $\Delta y \equiv y_r - y_e$.

The effect of a misalignment, consisting of a rotation and a translation, on the residuals can be calculated by considering the following. Let the parameters specifying the misalignment with respect to the nominal position of each module in its local frame be the linear offsets x_0, y_0, and z_0, and the rotations α_0, β_0, and γ_0, about the local axes. The expected coordinates transform passively according to:

$$\begin{bmatrix} x_{tr} \\ y_{tr} \\ z_{tr} \end{bmatrix} = \begin{bmatrix} 1 & \gamma_0 & -\beta_0 \\ -\gamma_0 & 1 & \alpha_0 \\ \beta_0 & -\alpha_0 & 1 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \\ 0 \end{bmatrix} - \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

The differences in coordinates of the hit in the transformed and the nominal frames are:

$$x_{tr} - x_e = -x_0 + \gamma_0 y_e \tag{1}$$

$$y_{tr} - y_e = -y_0 - \gamma_0 x_e \tag{2}$$

$$z_{tr} - 0 = -z_0 + \beta_0 x_e - \alpha_0 y_e \tag{3}$$

The transformed z coordinate of the hit with respect to the nominal frame is $-z_{tr}$. Considering the effect of track extrapolation for a point along the track at a distance δz from the module plane (Fig. 2 and 3), the recorded coordinates differ from the transformed coordinates of the expected hit by offsets δx and δy:

$$\delta x = \delta z \tan\psi, \tag{4}$$

$$\delta y = \frac{\delta z \cos\lambda}{\sin\theta\cos\psi}, \tag{5}$$

with ψ denoting the acute angle, in the global XY plane, between the track projection and the normal to the module plane cross-section, and λ denoting the angle, in the global RZ plane, between the module plane cross-section and the track. Hence, with $\delta z = -z_{tr}$, $x_r = x_{tr} + \delta x$ and $y_r = y_{tr} + \delta y$, the dependence of the residuals on the alignment constants is:

$$\Delta x = -x_0 + \gamma_0 y_e + (z_0 + \alpha_0 y_e - \beta_0 x_e)\tan\psi \tag{6}$$

$$\Delta y = -y_0 - \gamma_0 x_e + (z_0 + \alpha_0 y_e - \beta_0 x_e)\frac{\cos\lambda}{\sin\theta\cos\psi}, \tag{7}$$

where θ is the angle between the track and the global z-axis.

Procedure and Data

The data used in this study are generated by the ATLAS Offline Software Group at LBNL and simulates 48 hours of cosmic muons passing through ATLAS at a rate of $\sim$0.5 Hz [2]. The simulation includes noise, multiple scattering, and misalignment. Only the high momentum muon tracks ($p_t > 30$ GeV) are used for the purpose of alignment in order to minimize errors associated with multiple scattering.

In the current study it is assumed that the position and orientation of modules on L2 is precisely known and all modules on L0 and L1 can be aligned with respect to L2. Approximately 30% of the high momentum tracks pass only through L2, and are therefore unusable. For the rest of the tracks, a line is defined by the two most distant hits provided they reside on layer 2. Intersections of the line with the nominal module planes, i.e. expected positions, and residuals, are calculated. Figure 4 shows the distribution of the hits for the modules on L0 and L1. For modules with more than 50 hits, two independent approaches are employed to extract the alignment constants.

The **histogram-based approach** relies on the fact that, to a first order approximation, the residuals are linearly related to each of the alignment constants, as shown above. Therefore, given sufficient statistics, the constants for each module can be extracted by fitting histograms. Five histograms are created for each module: Δx vs. y_e yields $-x_0$ as its offset and γ_0 as its slope; Δy vs. x_e gives $-y_0$ as its offset; Δx vs. $y_e \tan\psi$ has slope z_0; Δx vs. $x_e \tan\psi$ and Δx vs. $y_e \tan\psi$ yield α_0 and $-\beta_0$, respectively. The better resolution in local x ensures a better resolution in the recovered alignment constants, therefore, all constants except y_0 are extracted from (6).

In the **χ^2-based approach**, the residuals are parametrized by the alignment constants according to (6) and (7), thus the merit function

$$\chi^2(x_0, y_0, z_0, \alpha_0, \beta_0, \gamma_0) = \sum \left(\frac{1}{\sigma_x^2}(-\Delta x - x_0 + \gamma_0 y_e + (z_0 + \alpha_0 y_e - \beta_0 x_e)\tan\psi)^2 + \right. \\ \left. + \frac{1}{\sigma_y^2}\left(-\Delta y - y_0 - \gamma_0 x_e + (z_0 + \alpha_0 y_e - \beta_0 x_e)\frac{\cos\lambda}{\sin\theta\cos\psi}\right)^2 \right) \tag{8}$$

can be formed, where the resolutions $\sigma_x = 0.02$ mm and $\sigma_y = 0.117$ mm are obtained in previous studies of the alignment of the barrel endcaps with actual cosmic data [1]. Minimizing (8) yields the alignment constants simultaneously and accounts for any correlations between them. The value of χ^2 is an assessment of the goodness of the fit.

Once the alignment constants are recovered, the residuals are corrected according to (6) and (7). The width of the distribution of the residuals after this correction is an indicator of the improvement in the spatial resolution of the PIXEL achieved through this alignment procedure.

Results

The following describes the results of the alignment of the 264 modules containing more than 50 hits. This requirement on the statistics necessary to align a module is based on considerations of the method performance as a function of number of hits per module (Fig. 4). The alignment constants for the selected modules are compared to the constants used as an input for the simulation. The correlation patterns obtained by both approaches are similar (Fig. 5). The constant resolutions - the standard deviations of the differences between the true and recovered values - for each approach are listed in Table 1.

Due to the orientation of the individual pixels in a module, the best spatial resolution is expected in local x, which is improved from 30 μm to 20 μm and 17 μm using histogram-based and the χ^2-based approaches, respectively (Fig. 6).The widths of the distributions of the residuals residing on the aligned modules before and after correction is summarized in Table

2, where the values are based on residuals corrected for x_0, y_0, z_0 and γ_0. The alignment constants α_0 and β_0 are not used for recalculation of residuals since their deviations are larger than the input values.

Discussion and Conclusion

The results for the constant resolutions obtained by both approaches agree well and, as expected, the χ^2 approach provides improved accuracy, as it takes into account the correlations between the alignment constants. Both approaches examine the limit of *infinite* statistics, simulated by applying the same artificial random misalignment set to all modules on L0 and L1, thus producing a large homogeneous data sample. The applicability of the model to the finite statistics (Fig. 4) available per module is then assessed. Details pertaining to each approach follow.

During testing, the **histogram-based** approach is applied to a sample with infinite statistics and the constants are recovered as expected. Similarly, the derived values agree with the input values when a set of random misalignments is applied to each module. Attempts at minimizing the effect of noise by eliminating tracks that may be artificial does not show any improvement. Finally, carrying out iterations by inputting the values of x_0, y_0, z_0 and γ_0, recalculating residuals and obtaining new alignment constants shows no improvement in the values recovered for α_0 and β_0. It is therefore concluded that this method is insensitive to misalignments in α_0 and β_0 on the order of 1 mrad, such as the ones used in the simulation.

Another investigated limitation to the resolution is the effect of the misalignment present in L2. The accuracy that can be achieved in determining the true misalignment constants in that case is limited since the line is defined by recorded hits on L2. The degree to which the results are affected depends not only on the magnitude of the misalignment, but also on any systematics present in the misalignment of the modules in L2. In general, multiple tracks, passing through the same module in an inner layer, will pass through different modules in L2, therefore, random misalignments in the L2 modules are averaged out. On the other hand, systematic misalignments can have a significant impact on the accuracy of the recovered constants. To isolate this effect, a perfectly aligned detector is considered, and a translation of 30 μm in local x is applied to all modules that belong to the upper part of L2. As expected, the error generated by this displacement is propagated through the layers with the largest effect in the upper part of L1 and the smallest effect in the lower part of L1 (Fig. 7).

In all $\boldsymbol{\chi^2}$**-related** calculations, a 2 mm residual cut is applied to curb errors due to noise, resulting in data reduction of up to 0.8%. Studies on the model performance examine possible systematics by considering the correlations between the input and the calculated output misalignment sets with infinite statistics. After examining ∼40 sets (Fig. 8) , any deviations from perfect correlations are attributed to either higher order contributions that are not factored in the model or to instability of the merit function in parameter space. The calculation is then tested for convergence upon iteration. It is determined that iterations do not improve the initially calculated constants and the algorithm is maximally efficient with infinite statistics.

To assess applicability of the approach to the real task, the calculations are performed with different constant set per module and with limited statistics. Upon iteration, the alignment constant resolutions do not improve, as expected. By considering the constant resolutions as functions of the number of hits per module, it is determined that less then 50 hits per module may be used with reasonable error penalty.

Subject to the limitations discussed, the described approaches have the potential of providing a computationally inexpensive method for the assessment of the alignment of the barrel modules. Significant improvement in the spatial resolution is demonstrated with the available data without a need for iterations. In addition, the derived alignment constant sets may be potentially used to examine the barrels for deformations by fitting surfaces (elliptical cylinders, one-sheeted hyperboloids, etc.).

In conclusion, the model is general enough to be applied to straight line tracks and need not be limited to cosmic muons. The first available data from ATLAS can be used for further testing of its usability. Further improvement of the algorithm, as well as a study of the alignment of the barrels, is subject to ongoing research.

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References

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[2] M. Scherzer *et al.*(1 July 2008). Alignment with cosmics. Presented at Pixel Meeting: Online and Offline Software. [Online]. Available : http://indico.cern.ch/materialDisplay.py?subContId=3&contribId=107&sessionId=19&materialId=slides&confId=22936

Tables

parameter	$x_0(\mu\text{m})$	$y_0(\mu\text{m})$	$z_0(\mu\text{m})$	α_0(mrad)	β_0(mrad)	γ_0(mrad)
with histograms	11.0±0.7	19±1	13.2±0.9	1.09±0.06	2.9±0.3	0.46±0.03
with χ^2	6.5±0.4	18±1	10.3±0.6	0.65±0.04	2.3±0.2	0.38±0.02

Table 1: Constant resolutions for the aligned 264 modules

approach	histograms		χ^2	
parameter	$\sigma_x(\mu\mathrm{m})$	$\sigma_y(\mu\mathrm{m})$	$\sigma_x(\mu\mathrm{m})$	$\sigma_y(\mu\mathrm{m})$
before	29.4±0.4	144.2±0.8	29.7±0.4	143.1±0.8
after	19.6±0.2	143.0±0.8	15.7±0.3	140.0±0.8

Table 2: Local resolutions for the aligned 264 modules before and after correction

Figures

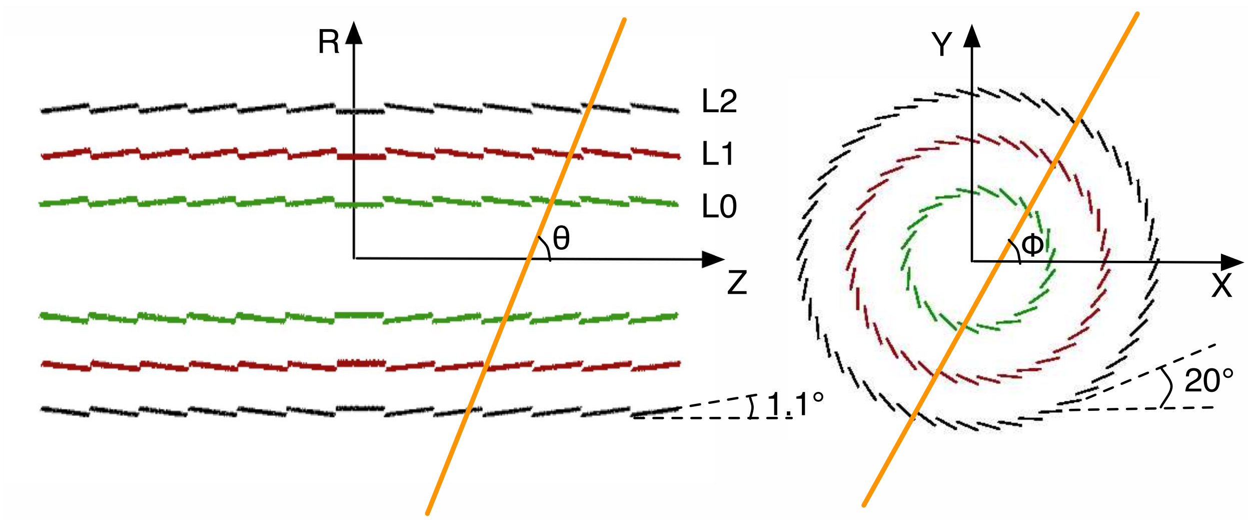


Figure 1: A schematic of the cross-sections of the barrel layers in the RZ and XY planes, illustrating the module arrangement. The orange lines represent a track in the RZ plane and the projection of the track in the XY plane.

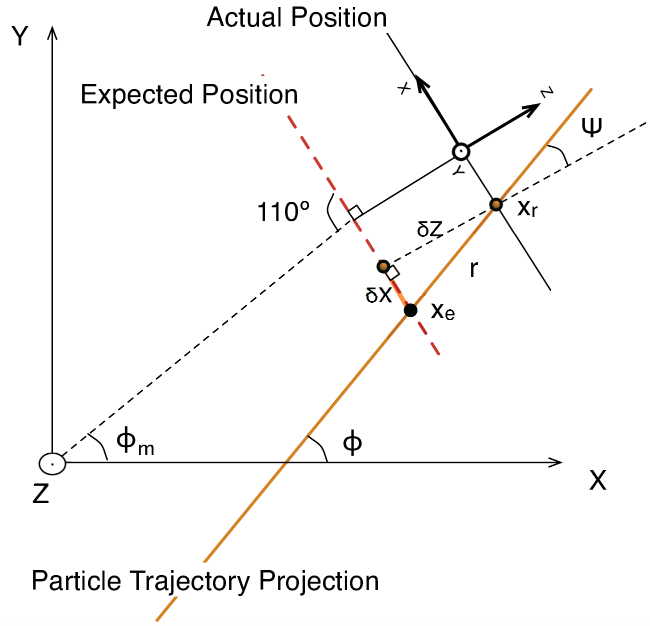


Figure 2: The offset δx due to a shift δz of the module in its local frame.

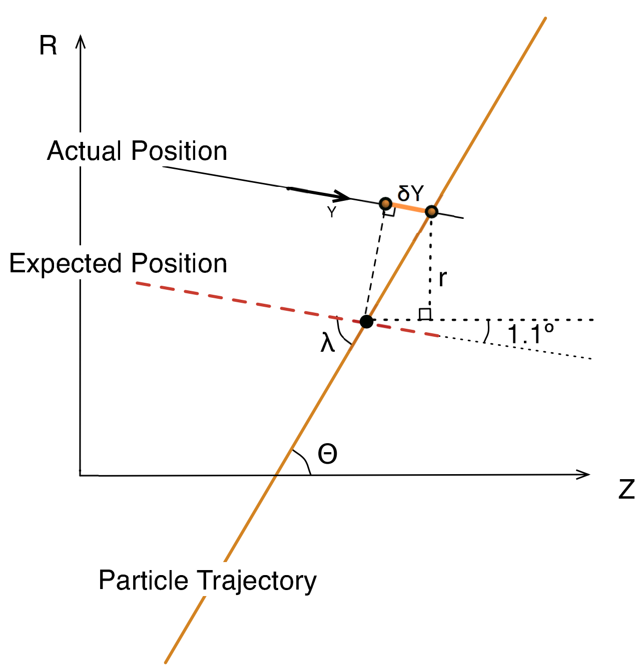
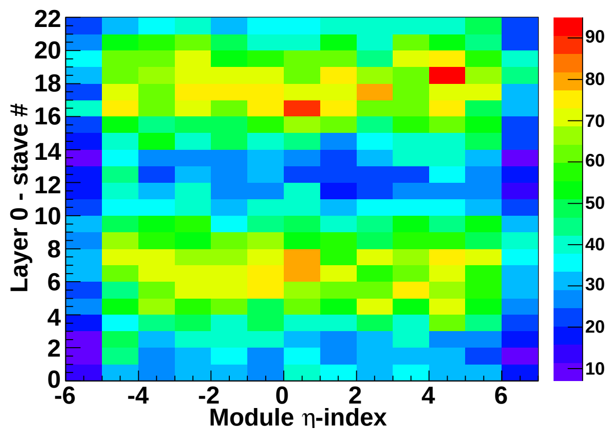


Figure 3: The offset δy due to a shift δz of the module in its local frame.



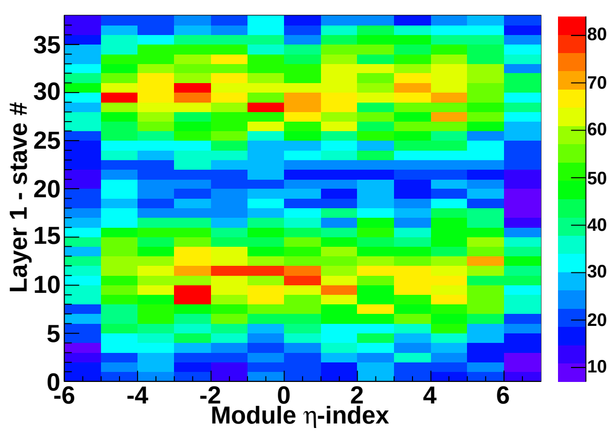


Figure 4: Distribution of hits for the modules on layers 0 and 1. Hit density is color-coded.

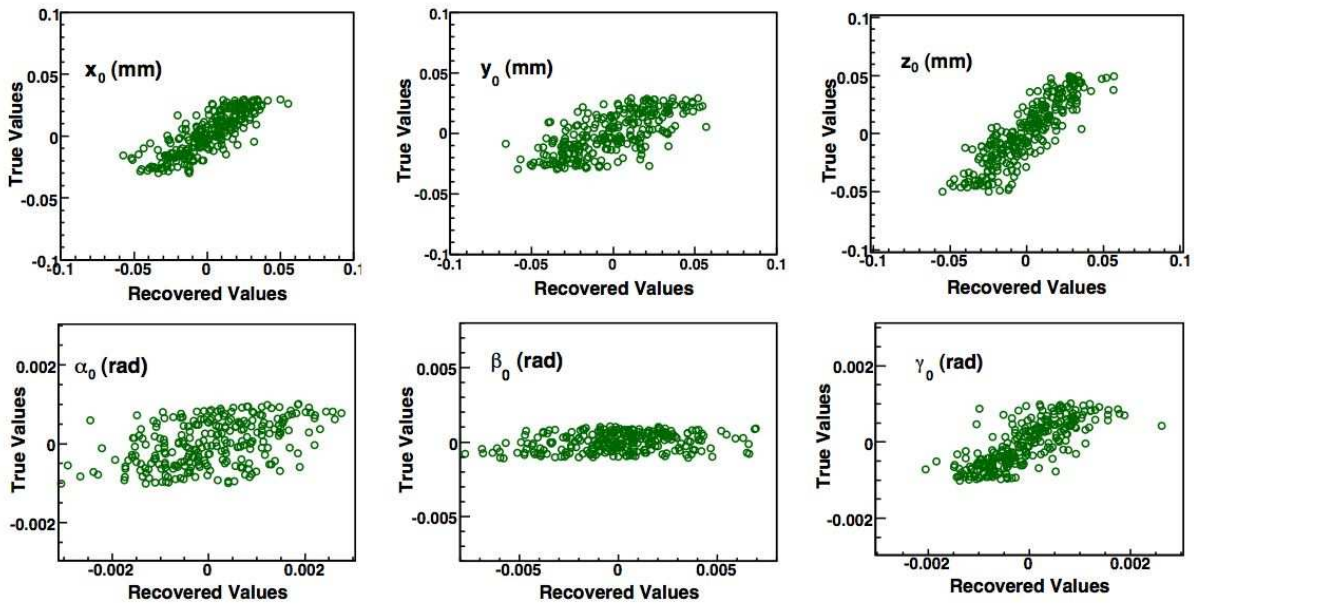


Figure 5: Correlations between the true and derived alignment constants for modules with more than 50 hits.

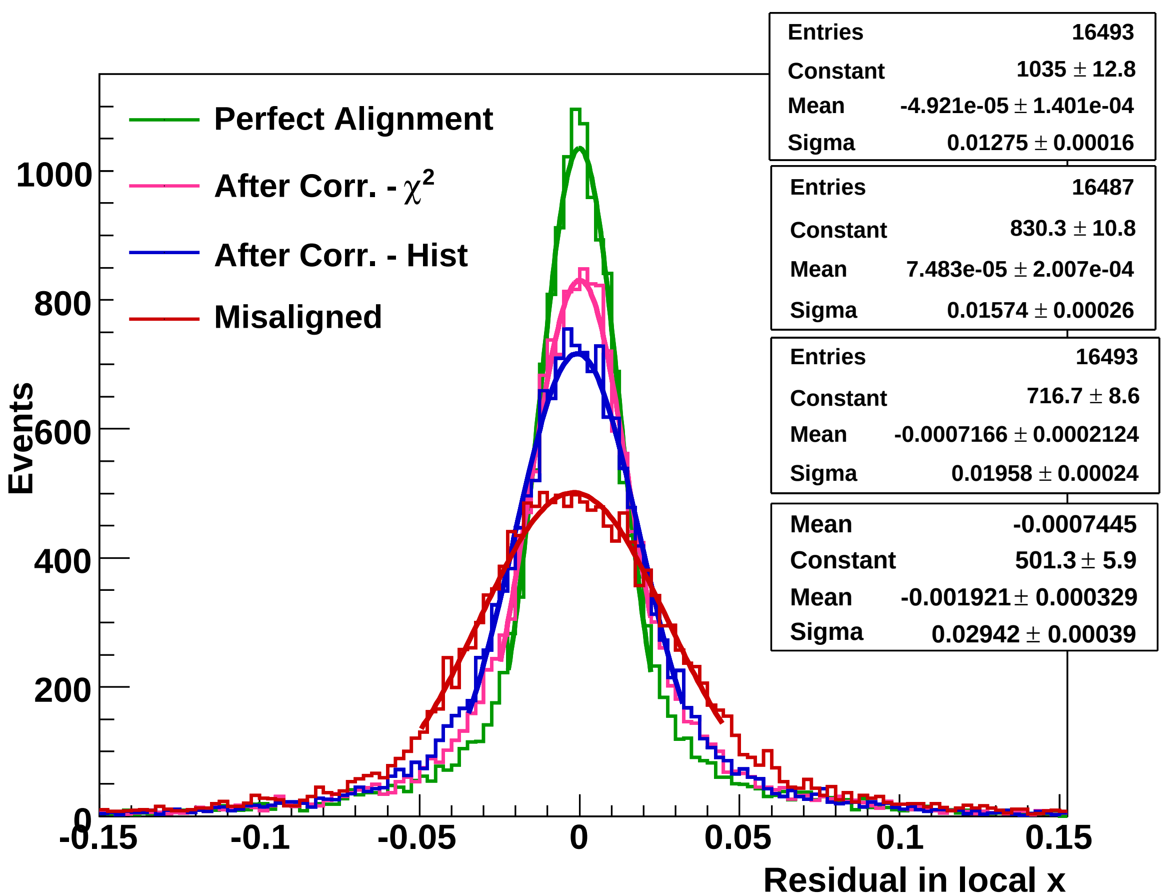


Figure 6: Improvement of the spatial resolution in local x.

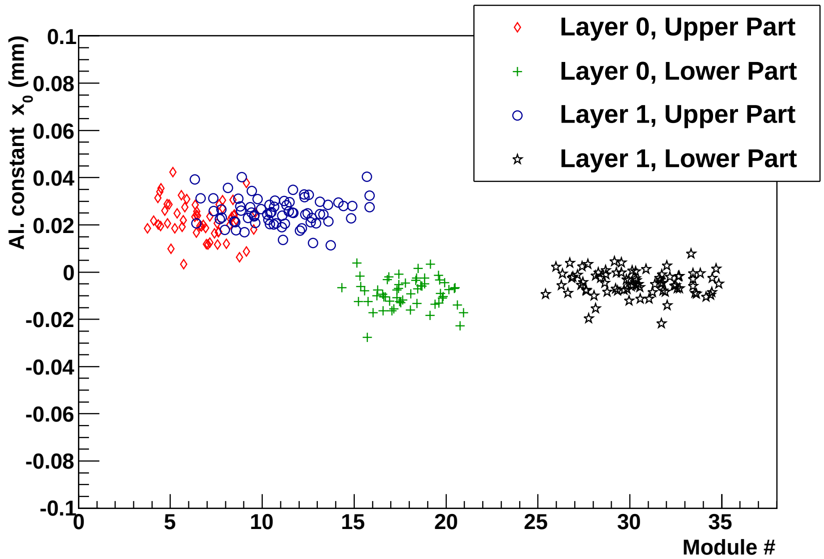


Figure 7: Effect of perturbation of L2 by 30μm in local x on the accuracy of the alignment constant x_0 for each module on layers L0 and L1. Color is used to emphasize the effect on different parts of each layer.

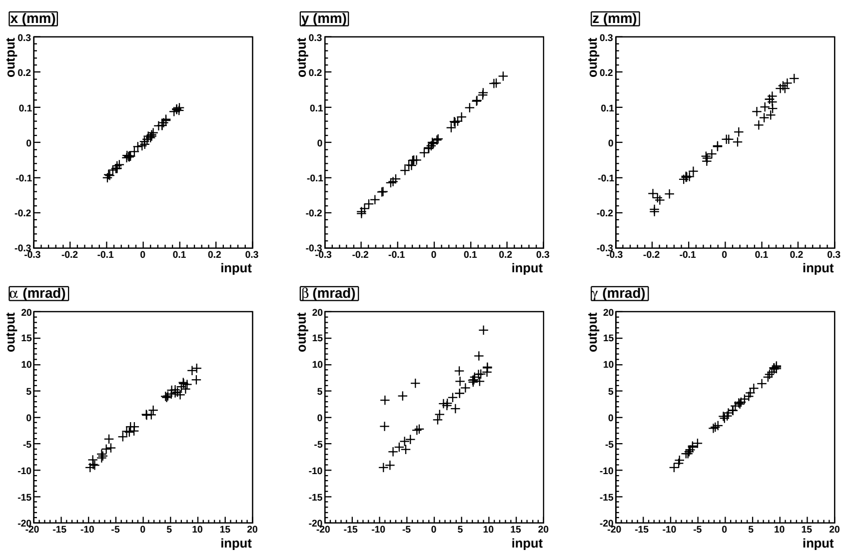


Figure 8: Correlations between the input and derived alignment constants with infinite statistics.